$$=L(1+x^2)-\frac{1}{x^2}L\,(1+x^2)+1+\frac{4}{x}(\tan^{-1}x-x).$$

$$\therefore S(+1)=4(\frac{\pi}{4}-1)+1=\frac{1}{3.1.2}-\frac{1}{5.2.3}+\ldots$$

$$\therefore \pi=3+\frac{1}{3}.\frac{1}{1.2}-\frac{1}{5}.\frac{1}{2.3}+\ldots$$

Also solved by G. B. M. Zerr, and V. M. Spunar.

339. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Prove that if $a_1 \nless 2$ and $a_n=a_{n-1}^2-2$, $\frac{1}{a_1}+\frac{1}{a_1\,a_2}+\frac{1}{a_1\,a_2\,a_3}+\ldots$ $=\frac{1}{2}[a_1-\sqrt{(a_1^2-4)}]$.

Solution by the PROPOSER.

Let the roots of the equation $x^2-a_1x+1=0$ be α and $1/\alpha$. Then

$$a_1=\alpha+\frac{1}{\alpha},$$

$$a_2=\alpha^2+\frac{1}{\alpha^2},$$

$$a_3=\alpha^4+\frac{1}{\alpha^4},$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

The series becomes

$$\frac{1}{\alpha+\frac{1}{\alpha}}+\frac{1}{\left(\alpha+\frac{1}{\alpha}\right)\left(\alpha^2+\frac{1}{\alpha^2}\right)}+\ldots+\left(\alpha-\frac{1}{\alpha}\right)\left[\frac{1}{\alpha^2-\frac{1}{\alpha^2}}+\frac{1}{\alpha^4-\frac{1}{\alpha^4}}+\frac{1}{\alpha^8-\frac{1}{\alpha^8}}\ldots\right]$$

Since $\displaystyle\frac{1}{\alpha^2-\frac{1}{\alpha^2}}=\frac{1}{2}\left(\frac{\alpha+\frac{1}{\alpha}}{\alpha-\frac{1}{\alpha}}-\frac{\alpha^2+\frac{1}{\alpha^2}}{\alpha^2-\frac{1}{\alpha^2}}\right)$,

each term of the series may be written as the difference of two fractions, *i. e.*

$$\frac{\alpha-\frac{1}{\alpha}}{2}\left[\left(\frac{\alpha+\frac{1}{\alpha}}{\alpha-\frac{1}{\alpha}}-\frac{\alpha^2+\frac{1}{\alpha^2}}{\alpha^2-\frac{1}{\alpha^2}}\right)+\left(\frac{\alpha^2+\frac{1}{\alpha^2}}{\alpha^2-\frac{1}{\alpha^2}}-\frac{\alpha^4+\frac{1}{\alpha^4}}{\alpha^4-\frac{1}{\alpha^4}}\right)+\ldots\right]$$

$$=\frac{a-\frac{1}{a}}{2}\left[\frac{a+\frac{1}{a}}{a-\frac{1}{a}}-\frac{a^{2^n}+a^{-2^n}}{a^{2^n}-a^{-2^n}}\right].$$

When $a>1$, the limit of this sum is

$$\frac{a-\frac{1}{a}}{2}\left(\frac{a+\frac{1}{a}}{a-\frac{1}{a}}-1\right)=\frac{1}{a}.$$

Therefore, the sum of the series is the smaller root of the equation $x^2-a_1x+1=0$, viz., $\frac{a_1-\sqrt{(a_1^2-4)}}{2}$.

NOTE.—This furnishes a very rapid method for finding square roots to a considerable number of decimals. Example.—Let $a=16$,

$$8-3\sqrt{7}=\frac{1}{16}+\frac{1}{16.254}+\frac{1}{16.254.645.64514}+\dots$$

and three terms of the series give $\sqrt{7}$ to 18 decimals, four terms will give 37 decimals, etc.

Also solved by S. Lefsehetz.

GEOMETRY.

365. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Given the coordinates of the four vertices of the tetrahedron, (x_1, y_1, z_1); (x_2, y_2, z_2); (x_3, y_3, z_3); (x_4, y_4, z_4): find volume and express it by a determinant.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and I. W. SMITH, A. M., Assistant Professor of Mathematics, North Dakota Agricultural College.

Let $\triangle$ =area of triangular base BCD of the tetrahedron, p the perpendicular from the vertex A on the base BCD, V its volume, $(x-x_2)\cos\alpha+(y-y_2)\cos\beta+(z-z_2)\cos\gamma=0$ the equation to the plane of BCD.

Then $\triangle\cos\gamma$ is the projection of the area BCD on the plane xy, and (x_2, y_2), (x_3, y_3), (x_4, y_4) are its angular points.